ClassPad 101

for ClassPad Version 3.00+

Lesson 20

Introduction to Differential Equations

Welcome

In this lesson we will explore the graphs of 1^{st}, 2^{nd} and n^{th} order differential equations. Differential equation graphs play an important role in many fields of engineering and physics. The graphs can help us visualize a solution to differential equations that may not be found otherwise.

Lesson Goals

- To be able to graph a 1^{st} order differential equation
- To be able to graph a 2^{nd} order differential equation
- To be able to graph an n^{th} order differential equation
- To understand what a slope field is

In Lesson 20, you will learn how to:

- Graph 1^{st}, 2^{nd} and n^{th} order differential equations
- Draw a solution curve by providing an initial value
- Draw a solution curve by plotting a point
- Solve a differential equation using dsolve
- Solve a differential equation using Laplace

Upon completion of this lesson, you will be able to answer the following questions:

1. How do you change the DiffEq editor to into a 1^{st} order differential equation?
2. What is the  used for?
3. What dialog do you open to change the Step size?
4. What is the IC page used for?

Time required

About 60 minutes.

Getting Started

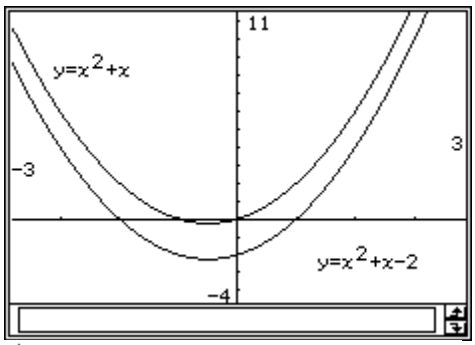
We will begin this lesson by considering what a differential equation is and what a slope field represents.

The simplest definition of a differential equation is an equation that involves a derivative. In calculus, you will learn how to find the derivative of a function and what it means. A "derivative" is represented by a form similar to dy/dx or y'. A "differential equation" is an equation that contains derivatives.

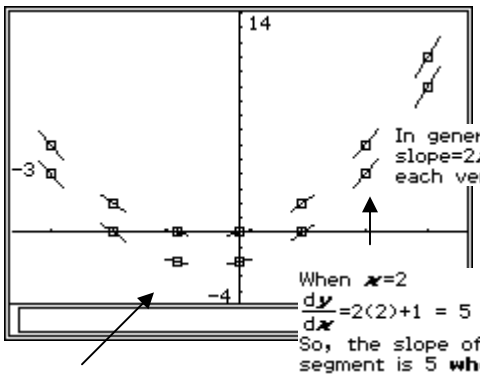
The simplest definition of a derivative is that it is the rate of change (slope) of the function at a given x-value. For example, if the function is linear, the slope is a constant rate of change and so the derivative is constant. Given y=3x+1, we know that the slope is 3 and so y'=3.

If the function is not linear, the derivative is very useful! It helps us find the rate of change at any individual point on the function.

To draw a slope field, we can use dy/dx or y' to find the slope of the original curve at a single x-value. We then draw a short segment about the x-value that has the estimated slope (dy/dx). If we do this enough times, we will be able to visualize the original curve!



$\frac{dy}{dx}=2x+1$ for both $y=x^2+x$ AND $y=x^2+x-2$!



When $x=2$
$\frac{dy}{dx}=2(2)+1 = 5$
So, the slope of the small line segment is 5 **whenever** $x=2$

When $x=-1$
$\frac{dy}{dx}=2(-1)+1 = -1$
So, the slope of the small line segment is -1 **whenever** $x=-1$

Can you visualize the original graph in the slope field? We can think of a slope field as what remains after erasing parts of the original!

Given a slope field (the ClassPad can do this); we can approximate a solution curve or the family of curves the original curve came from. All we need to do is connect together individual slopes to form a smooth pattern.

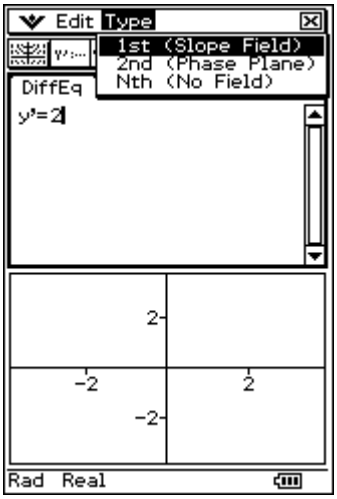
PART I

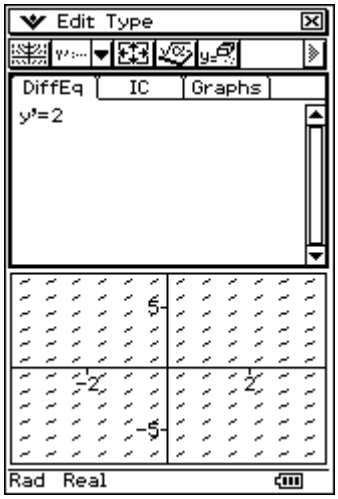
In this part, you will learn how to graph a 1st order differential equation and try to understand what the graph represents.

1. Graphing a Slope Field for a 1st Order Differential Equation

a. Open 

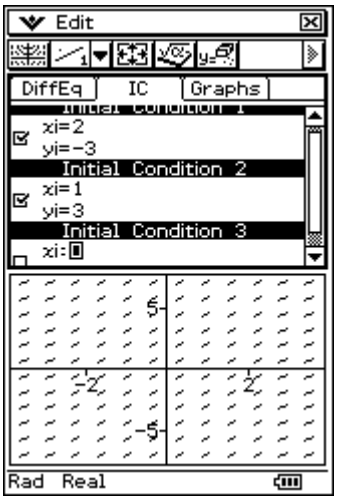
b. Select **Type / 1st (Slope Field)**
c. Following **y'**, input **2**
d. Press **EXE**
e. **Click**  to graph

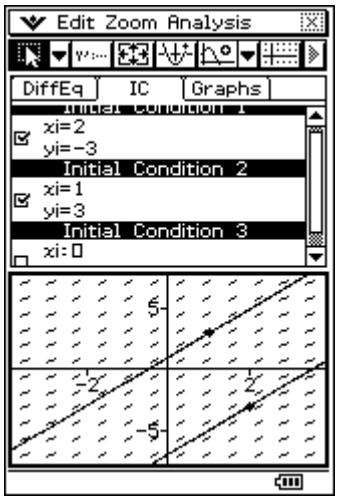




2. Inputting an Initial Condition Point using the IC Page

a. Click in the **DiffEq editor** window
b. Click the **IC** tab
c. Following **xi**, input **2**
d. Following **yi**, input **-3**
e. Press **EXE**
f. Following **xi**, input **1**
g. Following **yi**, input **3**
h. Press **EXE**
i. **Click**  to graph





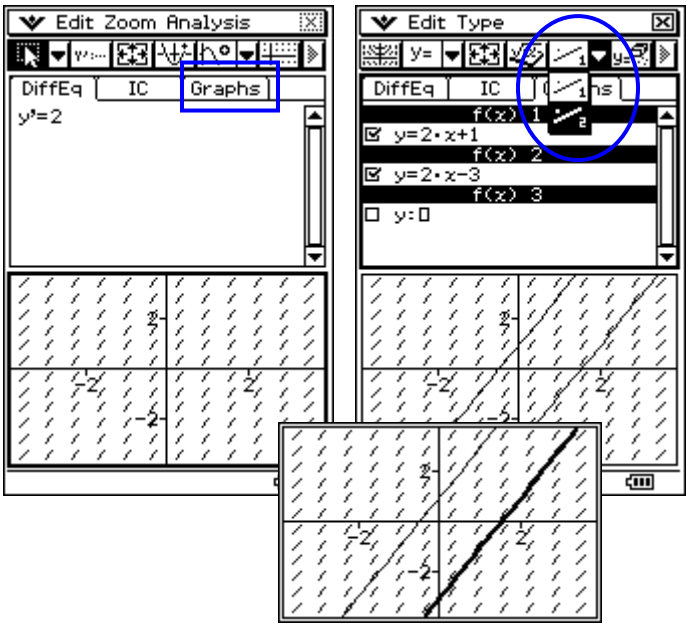
Notice that the solution curves we draw are linear! Does this make sense? Hmmm…

y'=2 (a constant) which implies that the original equation is a linear equation with a slope of 2. The original must be of the form y=2x+b. But, we would need additional information to find the exact value of b.

We could find b for (2,-3) and (1, 3).
For (2,-3) y=2x+b becomes -3=2*2+b -> b=-3-4=-7
What will b equal for (1, 3)? Guess by looking at the above graph!

3. Graphing Solution Curves by Guessing an Equation

a. Click in the **graph** window
b. Select **Edit / Clear All** to clear the **solution curve**
c. Click the **Graphs** tab of the graph editor
d. Following **y**, input **2x+1**
e. Press **EXE**
f. Click  to graph
g. Following the next **y**, input **2x-3**
h. Press **EXE**
i. **Click** the line containing **2x-3** in the Graphs page
j. Click n and select 
k. Click  to graph again



PART I
Practice Exercises

Before beginning the practice exercises, open a word document, type in the following information and then *save it as Lesson20 in your CASIO folder within My Documents*:

- Date: (enter today's date)
- To: (put your instructor's name here)
- From: (put your name here)
- Re: Lesson 20

1. Please begin by opening the DiffEqGraph application.
2. Click the **Graph** tab and select **Edit / Clear All**.
3. Click the **IC** tab and select **Edit / Clear All**.
4. Click the **DiffEq** tab and select **Edit / Clear All**.
5. Following **y'**, input **1 / y** and press **EXE**.
6. Display the graph.
7. With your equation and graph showing, get a **screen capture**. Paste it into your Lesson20 document (under a title of PART I).
8. Click the **IC** tab and input **x=1, y=2** and **x=1, y=-2** for initial conditions.

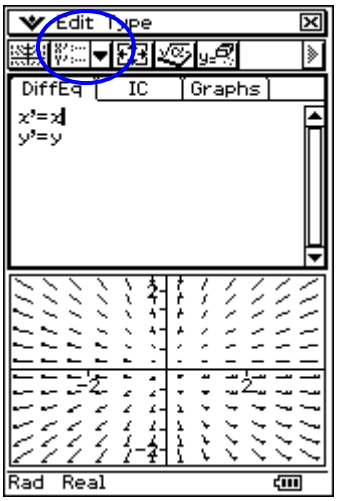
9. Display the graph with solution curves.
10. With your equation and graph showing, get a **screen capture.** Add two blank spaces following the first screen capture and then paste this one.
11. Click the **Graphs** tab. Following **y**, input **(2x)^(1/2)** and press **EXE**.
12. Place focus on the line containing (2x)^(1/2) and change the line style to be thicker.
13. Display the graph.
14. With your equation and graph showing, get a **screen capture.** Add two blank spaces following the last screen capture and then paste this one.
15. **Please** clear the DiffEq page, IC page and the Graphs page. Each needs to be done separately.

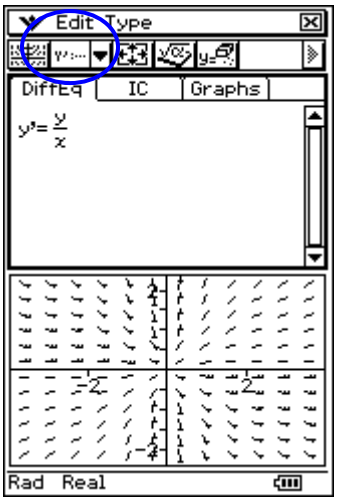
PART II

In this part, you will learn how to graph a 2nd order differential equation and how to change view window settings.

1. Graphing a Phase Field for a 2nd Order Differential Equation

a. Open DiffEqGraph and click in the **DiffEq editor** window
b. Select **Type / 2nd (Phase Plane)**
c. Following **x'**, input **x**
d. Following **y'**, input **y**
e. Press **EXE**
f. Click to graph
g. Click in the **DiffEq editor** window
h. Change the **Type** to **1st (Slope Field)**
i. Following **y'**, input **y / x**
j. Press **EXE** and graph again





In case you are wondering!

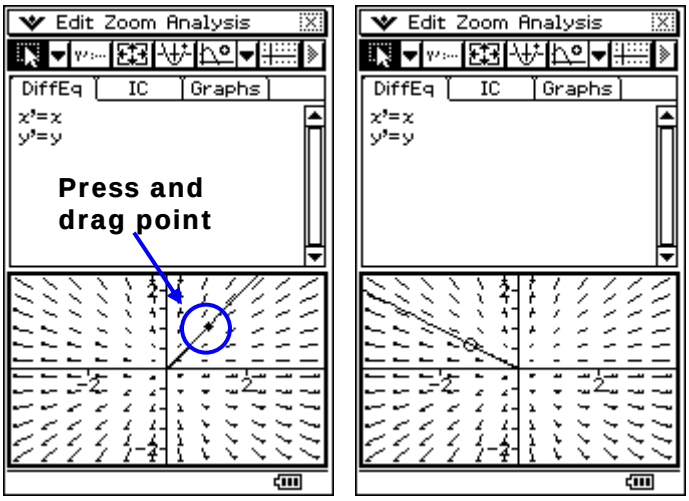
Note that $x'=\frac{dx}{dt}$, $y'=\frac{dy}{dt}$ and $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$

For $x'=x$ and $y'=y$,

we have $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{y}{x} \Rightarrow y'=\frac{y}{x}$

2. Graphing a Solution Curve through a Point

a. Click in the **DiffEq editor** window
b. Click the n on the toolbar and select
c. Great, our input was saved!
d. Click in the graph window to given it focus
e. **Click** (draw solution curve button)
f. Click in the graph window – a **solution curve** is drawn for you
g. Press and drag the point to move the solution curve

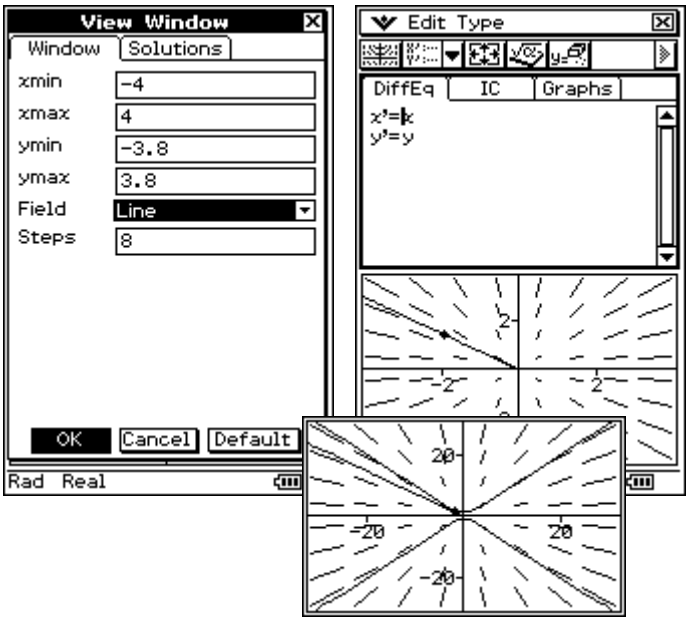


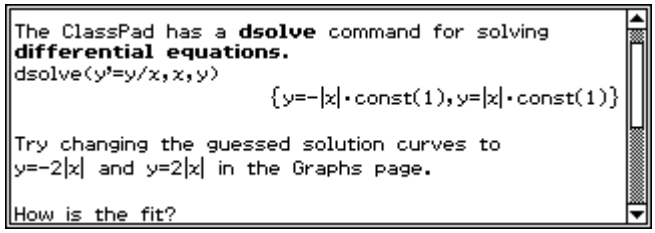
3. Exploring the View Window

a. Click in the **DiffEq editor** window
b. Click 6 to open the **View Window**
c. Change your values to match the ones shown
d. **Zoom** in and out using + and –

Can you guess a solution curve?

a. Click the **Graphs** tab
b. Input y=abs(x) and press **EXE**
c. Input y=-abs(x) and press **EXE**
d. Display the graph





PART II
Practice Exercises

1. Please begin by opening the DiffEqGraph application.
2. Click the **Graph** tab and select **Edit / Clear All**.
3. Click the **IC** tab and select **Edit / Clear All**
4. Click the **DiffEq** tab and select **Edit / Clear All**.
5. Change to the 2nd (Phase Plane) type.
6. Following **x'**, input **y-x** and press **EXE**.
7. Following **y'**, input **-2x** and press **EXE**.
8. Display the graph. Zoom until you like the graph.
9. With your equation and graph showing, get a **screen capture**. Paste it into your Lesson20 document (under a title of PART II).
10. Select **Analysis / Modify** to activate the *draw a solution curve* button.
11. Click in the graph window to draw a solution curve.
12. With your equation and graph showing, get a **screen capture.** Add two blank spaces following the first screen capture and then paste this one.
13. Click the *draw a solution curve* button ().
14. Click in the graph window to draw a solution curve again.
15. Resize the graph window.
16. With your graph showing, get a **screen capture.** Add two blank spaces following the last screen capture and then paste this one.
17. **Please** clear the DiffEq and IC pages. Each needs to be done separately.

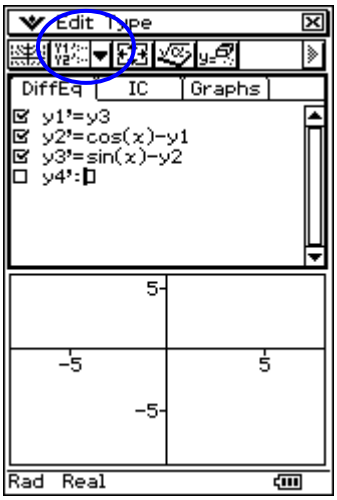
PART III

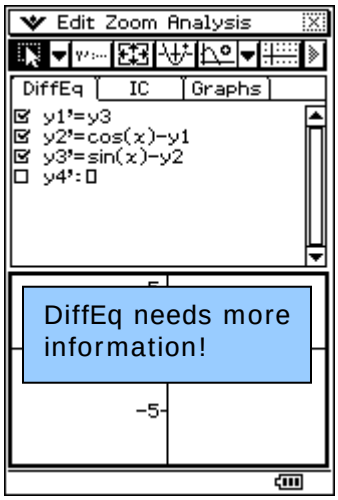
We have one more type of ClassPad differential equation to try. For this type, n^{th} order, we can only display solution curves based on initial conditions.

1. Inputting an n^{th} Order Differential Equation

a. Open DiffEqGraph and click in the **DiffEq editor** window
b. Select **Type / n^{th} (No Field)**
c. Following **y1'**, input **y3**
d. Press **EXE**
e. Following **y2'**, input **cos(x)-y1**
f. Press **EXE**
g. Following **y3'**, input **sin(x)-y2**
h. Press **EXE**
i. Click  to graph

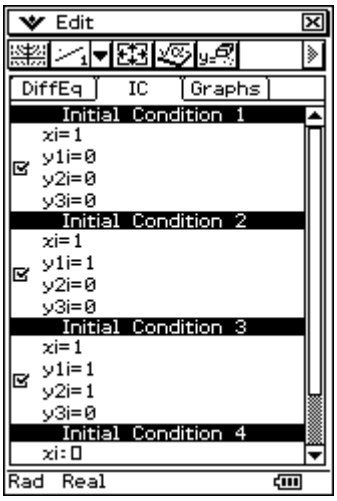
We need initial conditions!

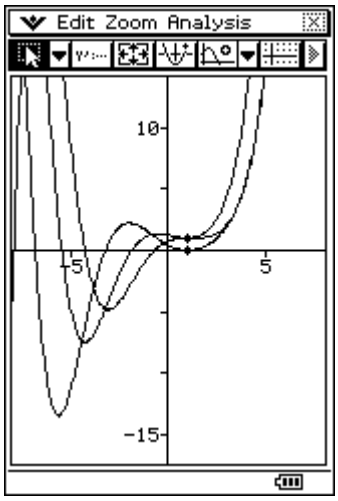




2. Inputting Initial Conditions

a. Click in the **DiffEq editor** window
b. Click the **IC** tab
c. Input the values for **Initial Condition 1**
d. Press **EXE**
e. **Click**  to graph
f. Input the values for **Initial Condition 2**
g. Press **EXE**
h. Input the values for **Initial Condition 3**
i. Press **EXE**
j. **Click**  to graph and resize the window





PART III

Practice Exercises

1. Open the eActivity named **L20_PartIII_a** in the **Lesson 20** folder.
2. Follow the directions inside the eActivity!
3. After you drag and drop all the required items, resize the graph window.
4. With the graph window showing, get a **screen capture**. Paste it into your Lesson20 document (under a title of PART III).
5. Note: You do not need to save your eActivities for Lesson 20.
6. Open the eActivity named **L20_PartIII_b** in the **Lesson 20** folder.
7. Follow the directions inside the eActivity.
8. After you drag and drop all the required items, resize the graph window.
9. With the graph window showing, get a **screen capture**. Add two blank spaces following the first screen capture and then paste this one.
10. Open the eActivity named **L20_PartIII_c** in the **Lesson 20** folder.
11. Follow the directions inside the eActivity.
12. After you drag and drop all the required items, resize the graph window.
13. With the graph window showing, get a **screen capture.** Add two blank spaces following the second screen capture and then paste this one.

PART IV

Reflection Exercises

You have just completed the twentieth lesson in ClassPad 101. Just one lesson to go! Please take a few moments to copy and paste the following three questions at the end of your Lesson20 document and answer them.

1. Approximately how long did it take you to complete this lesson?
2. Which activity did you enjoy the most?
3. Did you find any part of this activity difficult to follow? If so, which part? Also, how did you overcome the difficulty?

Assessment 20: Introduction to Differential Equations

- **Checkpoint**: Your word processed document, titled "Lesson20", should contain the following activities:
  1. Three screen captures from PART I.
  2. Three screen captures from PART II.
  3. Three screen captures from PART III.
  4. Three reflection questions with answers from PART IV.
- **Submit** your **Lesson20 document** to your instructor for grading. Once your lesson is submitted, your lesson for ClassPad 101 “Introduction to Differential Equations” is complete.