# ClassPad 101

*for ClassPad Version 3.00+*

LESSON 10

## Introduction to Matrices

### Welcome

In this lesson, we will cover some of the basic properties of a matrix. Matrices (the plural of matrix) provide us with an interesting way to store data in a two dimensional form. Remember that your main goal is to understand how to input matrices of various sizes; but I hope that you find the mathematical side of matrices interesting.

### Lesson Goals

- To understand how to name a matrix
- To become comfortable using matrix terminology
- To be able to add two matrices
- To be able to multiply two matrices
- To be able to read the solution to a system of equations from a matrix in reduced row echelon form

### In Lesson 10, you will learn how to:

- Create matrices of various sizes
- Find the dimensions of a matrix
- Add two matrices
- Multiply two matrices
- Multiply a matrix by a scalar
- Solve a system of equations
- Create an identity matrix

### Upon completion of this lesson, you will be able to answer the following questions:

1. When can two matrices be added?
2. When can two matrices be multiplied?
3. When two matrices are multiplied, what will the dimensions of the resulting matrix be?
4. How do we multiply a matrix by a scalar?
5. Name three ways to solve a system of two equations with two unknowns.
6. How can we verify that (x=-3, y=-4) is a solution to 3x – 5y = 11 using the ClassPad?

### Time required

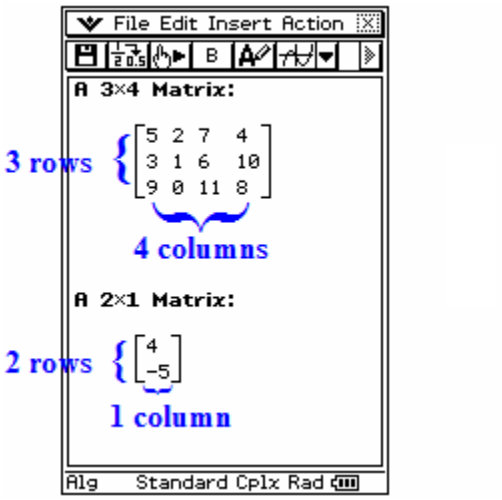
About 60 minutes.

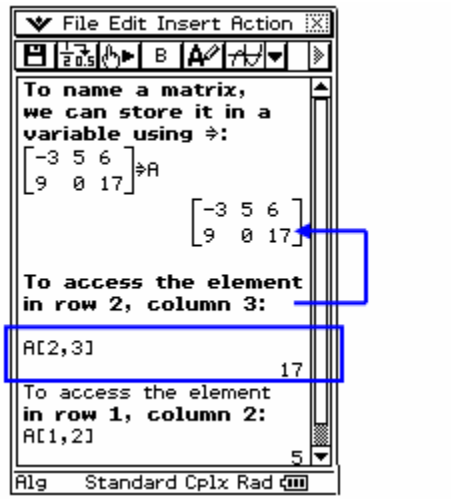
## Getting Started

What is a matrix? A matrix is a rectangular arrangement of values inside brackets. The "rectangular arrangement" is made up of rows and columns. We use the number of *rows by number of columns to name a matrix*. **For example, a 3x4 matrix has 3 rows and 4 columns.**

Each individual value inside a matrix is called an **element** of the matrix. If we have a matrix named **A**, **A[2,3]** means the individual value located at the *cross-section of row 2 and column 3*.

**Very Important**
When we name a matrix the **number of rows ALWAYS comes before the number of columns.** Just think RC.



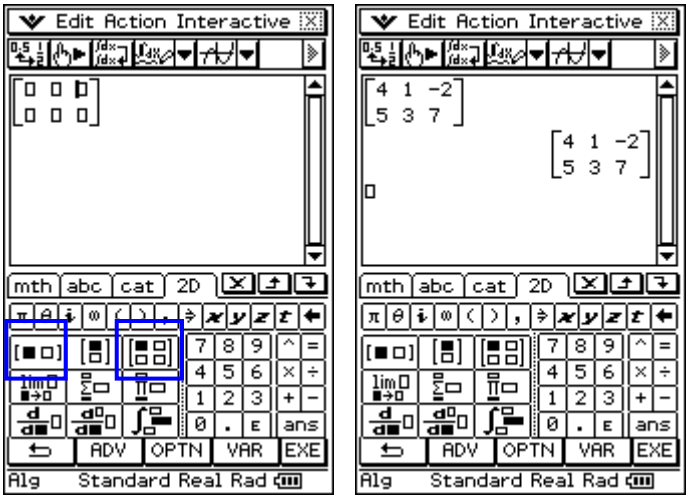



## PART I

In this part, we will practice creating matrices, adding matrices and multiplying a matrix by a number (called a *scalar*).

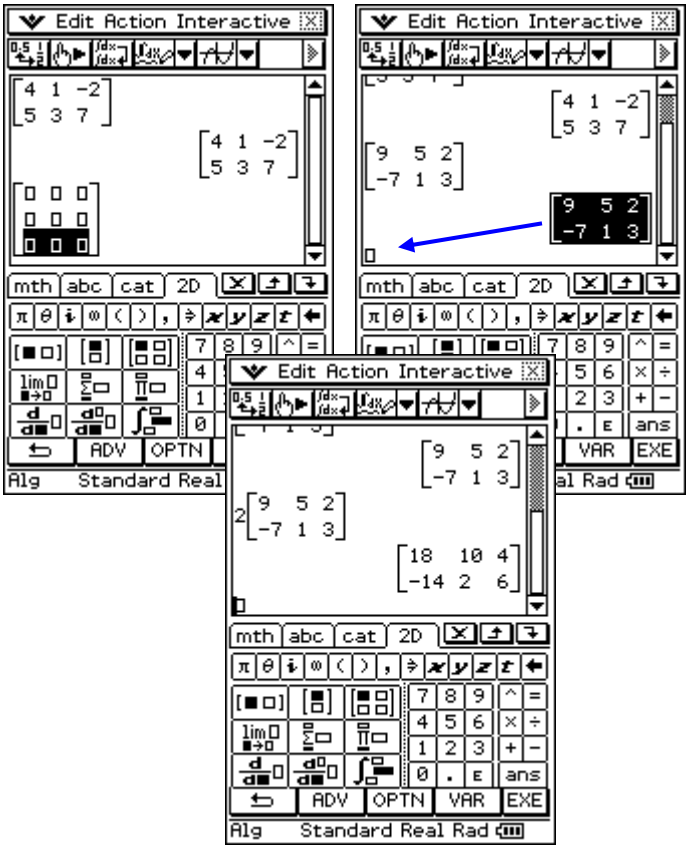
### 1. Inputting a 2x3 Matrix

a. Open J and then clear the window
b. Open the ClassPad's Keyboard (Ctrl + k)
c. Click on the **2D tab** and then click CALC
d. Input a **2x3 matrix (**Click 8 and then 6 to add one more column)
e. Input the numbers shown (***click inside*** *each box or use your* ***arrow keys***)




### 2. Inputting a 2x3 Matrix (Another Way) & Scalar Multiples

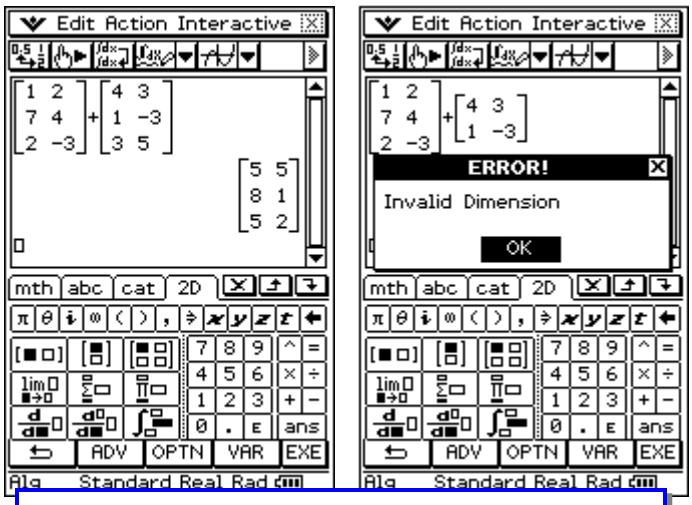
a. Click in the **small box below** your output
b. Click 8 twice (you now have a 3x3 matrix)
c. **Select** the **last row** and press the **delete key**
d. Input the numbers shown and the press **EXE**
e. **Select your answer** (output) and **drag it to the small box** on the next line
f. Using your **arrow keys**, **move the cursor** to the front of the matrix (the cursor will become **BIG**)
g. Input a **2** and press **EXE**
h. This is called ***scalar multiplication*** (we are multiplying by a constant value)
i. ***Notice each element inside*** the matrix ***was multiplied by 2***



### 3. Adding Two Matrices

When adding matrices, we add their corresponding elements.

a. Please clear your window
b. **Input** the two matrices shown (arrow keys will help you move around)
c. When finished, press **EXE**
d. Can you see how the matrices were added?
e. Select the last row of the second matrix and delete it
f. Press **EXE.** When **adding** matrices **they must be the same size** (same RxC numbers) or else it is not possible!




Matrices can be added *if and only if* they are of the same dimension.

## PART I
## Practice Exercises

**Before beginning the practice exercises, open a word document, type in the following information and then *save it as Lesson10 in your CASIO folder within My Documents*:**

- Date: (enter today's date)
- To: (put your instructor's name here)
- From: (put your name here)
- Re: Lesson 10

1. Please open the eActivity application.
2. Open the eActivity named **L10_PartI_a** in the **Lesson 10** folder.
3. Find the sum of the two given matrices inside the Verify window.
4. Once you find the sum, get a **screen capture**. Paste it into your Lesson 10 document (under a title of PART I).
5. **Save** your work as an eActivity named **L10_PartI_a**_your initials here.
6. Open the eActivity named **L10_PartI_b** in the **Lesson 10** folder.
7. In this exercise, you are asked to rewrite a single matrix as the sum of two matrices. Before beginning, open the Verify strip that contains an example so that you can see what you need to do.

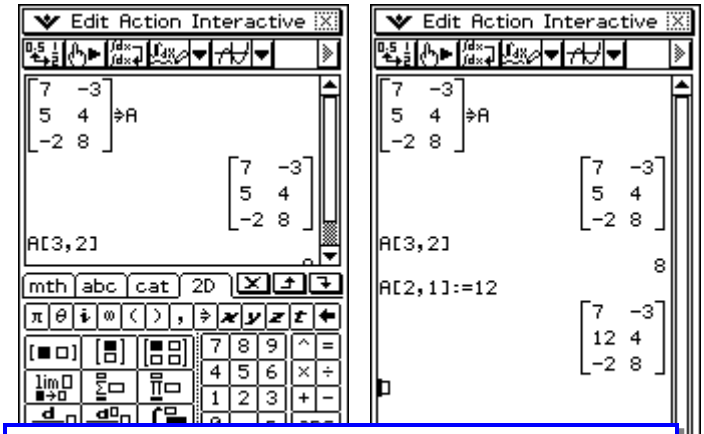
8. When you have your matrix rewritten (note that there can be many different answers), get a **screen capture.** Add two blank spaces following the first screen capture and then paste this one.
9. **Save** your work as an eActivity named **L10_PartI_b**_your initials here.
10. Open the eActivity named **L10_PartI_c** in the **Lesson 10** folder.
11. In this exercise, you will rewrite a single matrix as a scalar multiple times a matrix. Before beginning your exercise, open the Verify strip that contains an example.
12. When you have your matrix rewritten, get a **screen capture.** Add two blank spaces following the last screen capture and then paste this one.
13. **Save** your work as an eActivity named **L10_PartI_c**_your initials here.

## PART II

In this part, we will practice naming a matrix, accessing elements inside a matrix and multiplying matrices.

### 1. Naming a Matrix

a. Open Main and clear the window
b. Input a 3x2 matrix (as shown)
c. Using the store key (W), store the matrix in **A**
d. Press **EXE**
e. Access (get) the element in the 3rd row 2nd column by inputting **A[3,2]** and pressing **EXE**
f. Change the element in the 2nd row 1st column to **12** by inputting **A[2,1]:=12** and pressing **EXE**




Note: Inputting **A[2,1]:=12** does the same thing as inputting **12** W **A[2,1].** Both store a 12 in the variable location A[2,1].

**Is 5x3 = 3x5? Yes!** Multiplication with numbers is commutative. **BUT**, multiplication with matrices is different (much different).

**We can only multiply two matrices if they can be placed so that their "inner" dimensions are the same. For example,**

We ***can*** multiply $\mathbf{A_{2x3}}$ **x** $\mathbf{B_{3x4}}$ (Same "inner" dimensions)

We ***cannot*** multiply $\mathbf{B_{3x4}}$ **x** $\mathbf{A_{2x3}}$ (Different "inner" dimensions)

When we multiply $\mathbf{A_{2x3}}$ **x** $\mathbf{B_{3x4}}$ *the* ***result*** *is a* ***2x4 matrix.***

("outer" dimensions)

We ***can*** multiply $\mathbf{C_{2x2}}$ **x** $\mathbf{D_{2x3}}$ (Same "inner" dimensions)

We ***cannot*** multiply $\mathbf{D_{2x3}}$ **x** $\mathbf{C_{2x2}}$ (Different "inner" dimensions)

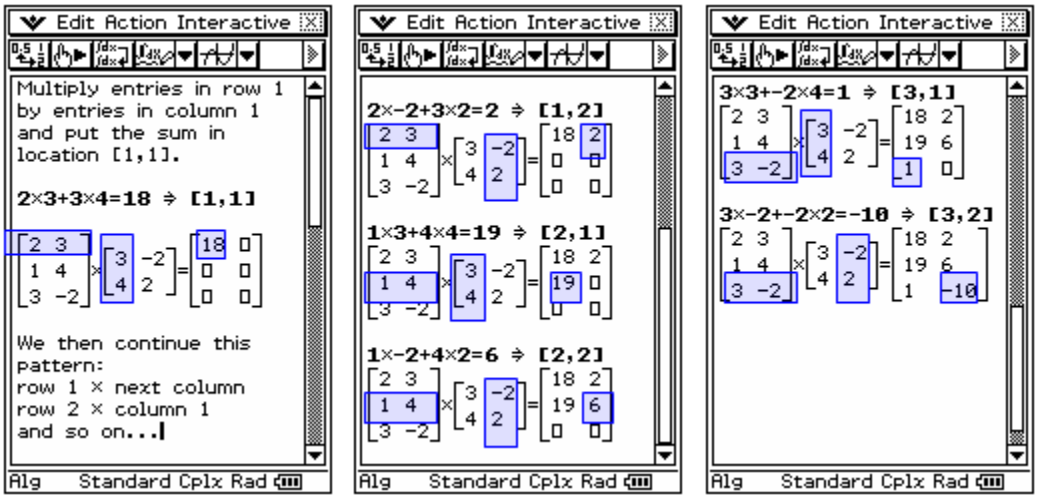
When we multiply $\mathbf{C_{2x2}}$ **x** $\mathbf{D_{2x3}}$ *the* ***result*** *is a* ***2x3 matrix.***
("outer" dimensions)

We **cannot** multiply $\mathbf{A_{3x2}}$ **x** $\mathbf{B_{1x4}}$ even if we rearrange them.

Just remember that you can only multiply $\mathbf{A_{RxC}}$ **x** $\mathbf{B_{RxC}}$ when the # of columns in the first matrix is equal to the # of rows in the second matrix.

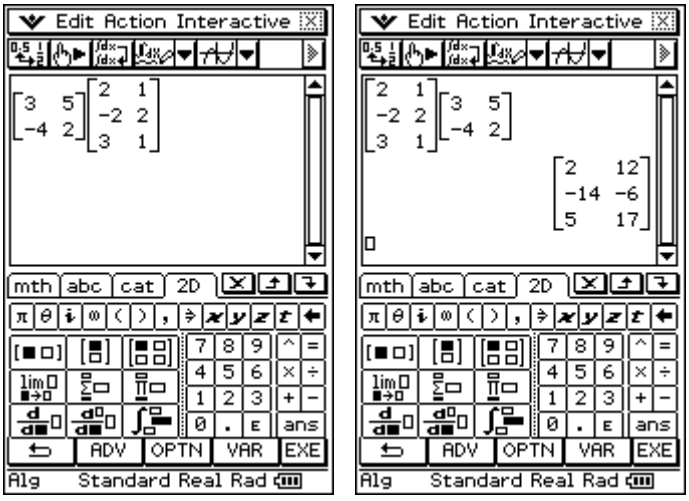
When we multiply two matrices, the ***resulting matrix dimensions*** are the same as the "outer" dimensions.

**We multiply two matrices as follows:**



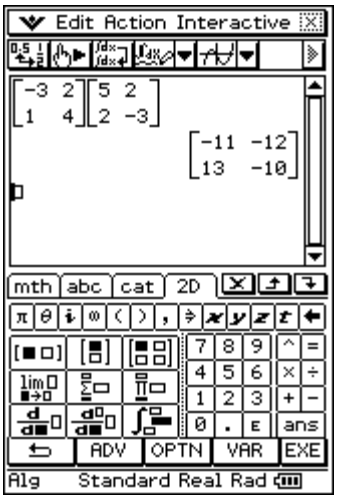

## 2. Multiplying Matrices

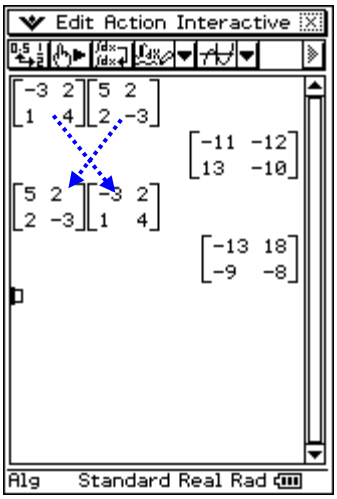
a. Clear your Main window
b. **Input** the **2x2** and **3x2** matrices shown
c. Press **EXE** (notice the column # of the 1st is 2 and row # of 2nd is 3 ⇒ *cannot* multiply)
d. **Select** the **2nd matrix** and drag it to the **front**
e. **Select** the **3rd matrix** and **delete it**!
f. Press **EXE** again (notice the result is a **3x2** matrix)



## 3. Square Matrices (AxB does not always equal BxA)

a. Clear your Main window again
b. **Input** the **2x2** matrices shown and press **EXE**
c. Drag and drop the **second matrix** to the **next input line**
d. Drag and drop the **first one so it is now second**
e. Press **EXE** (**Notice** the results are **different**)



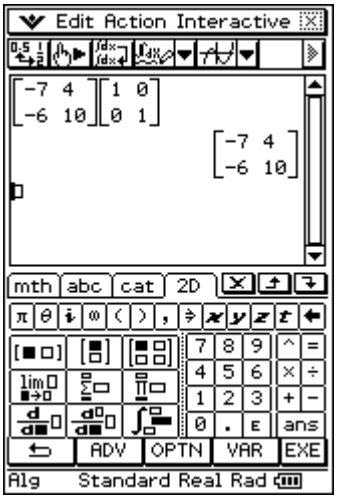



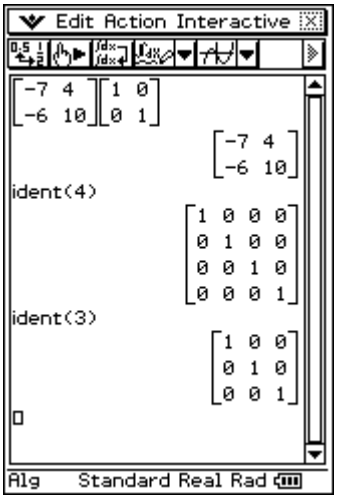

## 4. Special Square Matrices (Identity Matrix)

The multiplication identity element for real numbers is the number 1 because 1 times any number is equal to the number. Square matrices have an identity matrix and the idea is similar.

a. Clear your Main window again
b. **Input** the 2x2 matrices shown and press **EXE**
c. **Notice** your answer is identical to the 1st matrix
d. ***Try*** multiplying these by hand to see why this happened (interesting)
e. Type in **ident(4)** and press **EXE**
f. Type in **ident(3)** and press **EXE**

*Instead of typing in ident(, you can open the **Interactive menu** and select **Matrix-Create/ident**




☺**Very Helpful Hint:** When you need to copy to a place not showing in the work window, try using Edit/Copy and then Edit/Paste or Ctrl+c and Ctrl+v. It can be difficult to drag an equation when there is a long scroll bar present!

# PART II
# Practice Exercises

1. Please open the eActivity application.
2. Open the eActivity named **L10_PartII_a** in the **Lesson 10** folder.

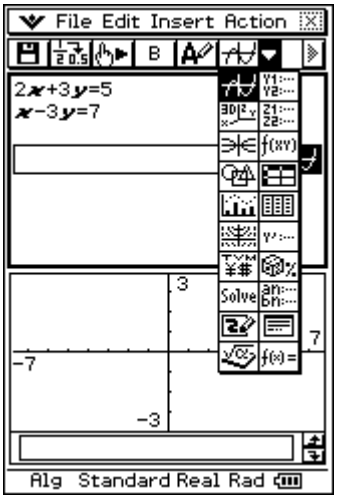
3. Notice that three different matrices are assigned to A, B and C. Think about their dimensions and decide which two pairs of matrices can be multiplied together. Hint: Write the dimensions on a piece of paper.
4. Scroll down and input your first pair and then press **EXE.** Make sure you use upper case letters and a times sign between them.
5. With the product of your first pair showing, get a **screen capture**. Paste it into your Lesson 10 document (under a title of PART II).
6. Find another pair that you can multiply together.
7. With the product of your second pair showing, get a **screen capture.** Add two blank spaces following the first screen capture and then paste this one.
8. **Save** your work as an eActivity named **L10_PartII_a**_your initials here.
9. Open the eActivity named **L10_PartII_b** in the **Lesson 10** folder.
10. Follow the directions inside the eActivity.
11. When you have your matrix multiplied by the identity matrix, get a **screen capture.** Add two blank spaces following the second screen capture and then paste this one.
12. **Save** your work as an eActivity named **L10_PartII_b**_your initials here.

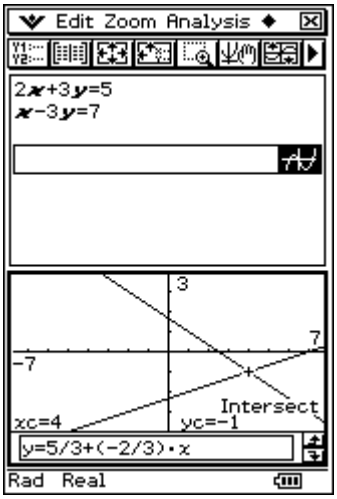
## PART III

One of the topics covered in Algebra is solving systems of equations. In this part, you will learn how to solve a system of two equations in three different ways and also a system of three or more equations in two different ways.

### 1. Solving a System of Equations Graphically

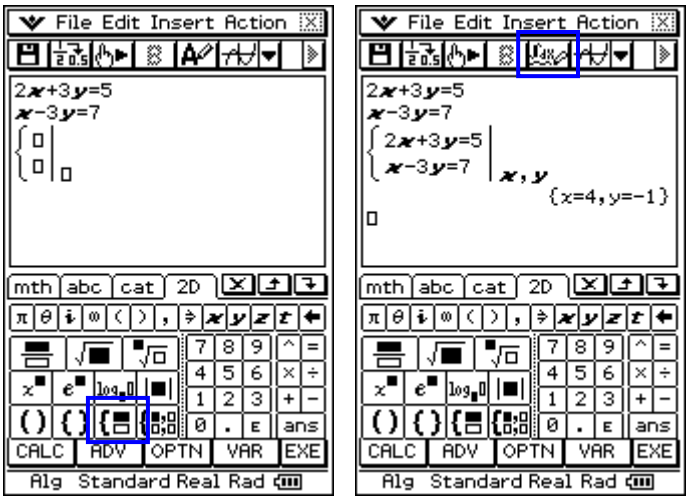
a. Open eActivity and **type in** the equations shown
b. **Insert** a **Graph** window strip
c. **Select** the **1st equation** and **drag** it to the **Graph window**
d. **Select** the **2nd equation** and **drag** it to the **Graph window**
e. From the Graph window's menu, select **Analysis / G-Solve / Intersect**
f. Note x=4, y=-1 Remember this point!



## 2. Solving a System of Equations using 2D Math

a. Keep your equations from step 1
b. Delete the Graph strip by clicking **inside the strip** and selecting **Edit / Delete Line**
c. Open the keyboard
d. Click on the **2D tab**
e. Click the System of Equations symbol ( )
f. Drag each equation to a box inside
g. Type your variables: **x,y** into the outside box
h. **Change** the line to a **math line** and press **EXE**



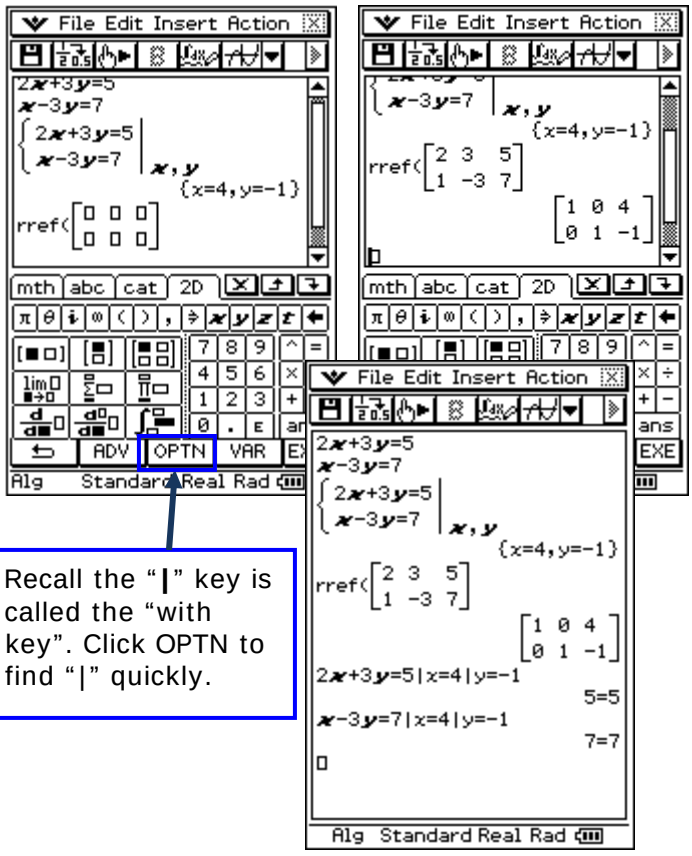

## 3. Solving a System of Equations Using rref & Matrices

a. Type in **rref(**
b. Input a **2x3 matrix** template
c. **Type in** the coefficients of your system of equations
d. Press **EXE**

*****Notice the x-coefficients were in column 1; y-coefficients were in column 2.

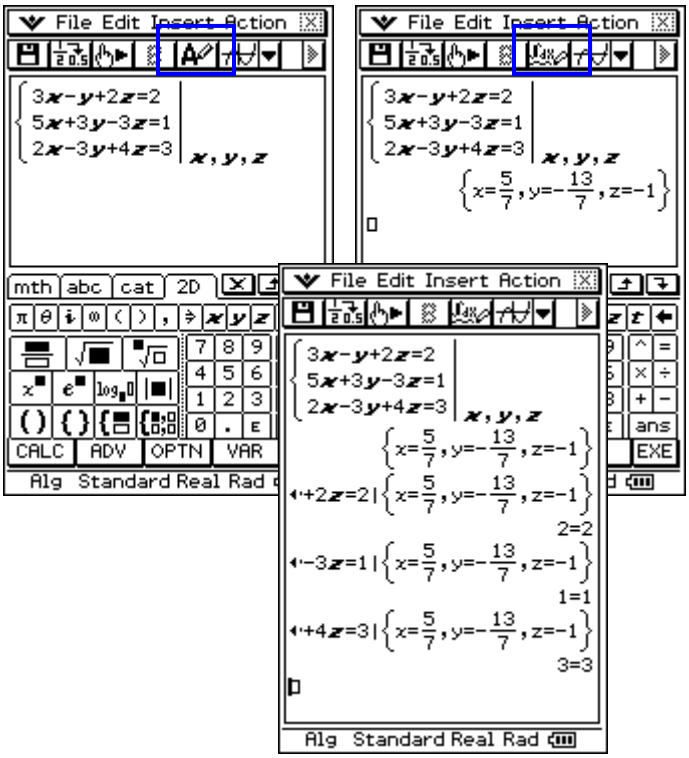
*Row 1 of the answer can be read **1x=4 or x=4**, row 2 can be read **1y=-1 or y=-1***

☑ Let's check
☑ Drag and drop the 1st equation to the next open math line
☑ Following the equation **type** |x=4|y=-1
☑ Press EXE (5=5 is true so it checks)
☑ Repeat with the 2nd equation



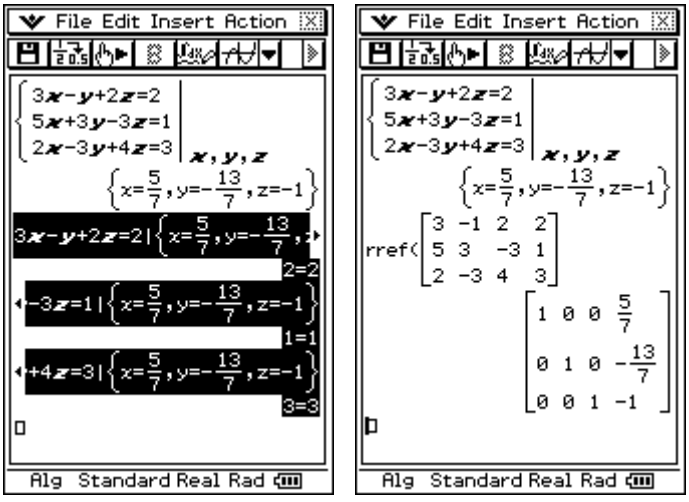
## 4. Solving a System of Equations with Three Unknowns

a. Clear your eActivity window
b. Click on the **2D tab**
c. Click the ⮌ button
d. Click the System of Equations symbol ( ) **twice**
e. **Type in** the equations shown
f. **Type in x,y,z** into the outside box
g. **Change** to a **math line** and press **EXE**
   - ☑ An easy way to check:
   - ☑ Select the first equation and drag to the next math line
   - ☑ Input | (the with key)
   - ☑ Select your answer and then drag & drop it following the | key
   - ☑ Repeat for equations 2 and 3




## 5. Solving a System of Equations with Three Unknowns and rref

a. **Select** the three lines used for checking and press the **delete key**
b. Type in **rref(**
c. Following **rref(**, input a **3x4 matrix** …this is fun
d. **Type in** the coefficients of your system of equations
e. Press **EXE**
f. Can you read the solution from the resulting matrix?




*Note: We cannot find the solution graphically on the ClassPad because we can only graph one 3-D graph at a time. For the 1st equation you could graph z=-3x+y+2 in a 3-D graph window and a flat plane would be drawn. If we could graph the other two planes in the same window, they would all intersect at just one point: (x=5/7, y=-13/7, z=-1).

## PART III
## Practice Exercises

1. Please open the eActivity application.
2. Open the eActivity named **L10_PartIII_a** in the **Lesson 10** folder.
3. Read the eActivity and then complete the exercise part.
4. When finished, get a **screen capture** with the **Complete: x= y= part showing**. Paste it into your Lesson 10 document (under a title of PART III).
5. **Save** your work as an eActivity named **L10_PartIII_a**_your initials here.
6. Open the eActivity named **L10_PartIII_b** in the **Lesson 10** folder.
7. Follow the directions inside the eActivity. Drag & drop will save time ☺.
8. **With your solution showing**, get a **screen capture.** Add two blank spaces following the first screen capture and then paste this one.
9. **Save** your work as an eActivity named **L10_PartIII_b**_your initials here.
10. Open the eActivity named **L10_PartIII_c** in the **Lesson 10** folder.
11. Follow the directions inside the eActivity.
12. **With your solution showing**, get a **screen capture.** Add two blank spaces following the last screen capture and then paste this one.
13. **Save** your work as an eActivity named **L10_PartIII_c**_your initials here.

## PART IV

## Reflection Exercises

You have just completed the tenth lesson in ClassPad 101. A great deal of mathematics that can be done using matrices; we just touched the surface. Please take a few moments to copy and paste the following three questions at the end of your Lesson 10 document and answer them.

1. Approximately how long did it take you to complete this lesson?
2. Which activity did you find particularly useful?
3. Did you find any part of this activity difficult to follow? If so, which part? Also, how did you overcome the difficulty?

## Assessment 10: Introduction to Matrices

- **Checkpoint**: Your word processed document, titled "Lesson10", should contain the following activities:
  1. Three screen captures from PART I.
  2. Three screen captures from PART II.
  3. Three screen captures from PART III.
  4. Three reflection questions with answers from PART IV.
- **Submit** your **Lesson 10 document** to your instructor for grading.